European Journal of Engineering and Environmental Sciences | *ISSN pending*
Published by DEQE Publications| *https://ejees.deqepub.org*
4 Rhindmuir Dr, Baillieston, Glasgow G69 6ND, UK
deqepub@gmail.com; enquiry@deqepub.org

Determining Mathematical Simulations for a Hydropower Turbine Governor

Okoro Chibuike

Department of Electrical Electronic Engineering

Enugu State University of Science and Technology, (ESUT), Nigeria.

Publication Process	*Date*
Received	***February 1st, 2021***
Accepted	***February 15th, 2021***
Published	***February 28th, 2021***

ABSTRACT

This paper aims at ensuring the development of mathematical models and equations for use in calculating parametric values for hydropower systems. Specific mathematical models and equations necessary for solving parametric related problems as encountered in this research are carefully presented. Meta-analysis research design was used. Synoptic reasoning was also used to develop a new and better understanding of the research problem. Actual values of various parameters, namely, Flow rate, (Q), Velocity, (V), Penstock length, (L), Output Power, (P), and Gate opening, (G), are calculated using the developed mathematical values. Different reservoir water heads are used to calculate actual values of these parameters. The over-all import and relevance of these developed models are seen in the final results obtained in this paper.

Keywords: *Hydropower system, Mathematical model, Meta-analysis, Synoptic reasoning, Flow rate, Penstock, Gate opening*

1. Introduction

A mathematical model is a description of a system by the use of mathematical concepts and language. This process is known as mathematical modeling. Specifically, it helps to explain a system and to study the effects of its different components thus, making dependable predictions about the system's behavior. There are many forms of mathematical models namely; dynamical systems, statistical models, differential equations or even game theoretic models logic models may also be included in mathematical models. Engineers often use mathematical models to analyze any system which needs to be controlled or analyzed. A set of variables and a set of equations meant to establish relationships between the variables are usually described using mathematical models.

In this paper, relevant mathematical models for use in the calculation and eventual determination of the parametric values for the design of a hydropower turbine governor are deduced and presented for use. These models could be manipulated to arrive at different parametric values for use in the design and construction of different specifications of hydropower systems. In addition to already known mathematical relations for this purpose, mathematical models and equations are developed in this paper for various parameters of the hydropower system namely; Flow rate, (Q), Velocity, (V), Penstock length, (L), Output Power, (P), and Gate opening valve, (G). Subsequently, specific parametric values are calculated for use in a design environment.

Review of Related Works

In their work titled, A Simple Dynamical Model of Hydropower Plant: Stability and Oscillations, G. A. Leonov et al, (2015), carried out extensive investigations into oscillations in hydropower plants. These consist of synchronous generator, hydraulic turbine and speed governor. A mathematical model of the hydropower unit was based on the equations of Park-Gorev for synchronous generator which was discussed in details in (Adkins, 1962 and Boldea, 2006). Both turbine and governor equations were treated. Following the established equations, oscillations in zones, which were not recommended for operation were found and were consistent with full-scale test results carried out for hydropower units.

Runfan Zhang, et al, (2015), in Non-Linear Predictive Control of a Hydropower Plant Model, introduced simple methods for finding an appropriate terminal penalty function and also proved its effectiveness. Lyapunov function was used to prove the input to state stability of the controlled system. Presentation of a six-dimensional model of the hydropower system which includes the hydro-turbine, the penstock system, the generator system and the hydraulic servo system, followed. The method produced desired stability. A strategy to combine the nonlinear predictive control method with other methods to further facilitate the application of nonlinear predictive control method was proposed by the authors.

In their paper, **Fixed-Time Stability of the Hydraulic Turbine Governing System, Caoyuan Ma, et al, 2018**, studied the problem of fixed-time stability of hydraulic turbine governing system with nonlinear model of elastic water hammer. According to the paper, a new fixed-time control strategy was proposed in order to control and improve the quality of hydraulic turbine governing system. This was capable of stabilizing the water turbine governing system within a fixed time. The settling time of the fixed-time control scheme could be adjusted to the required value regardless of the initial conditions compared to the finite-time control strategy where convergence rate depended on initial state. Mathematically based initial results showed the effectiveness and superiority of the fixed-time control over the finite-time control.

The reviews presented showed the applicability of mathematical models in carrying out scientific research and calculations leading to realization of parametric results needed in the design of prototype models. Thus, the deduced mathematical models are meant to be of immense use in carrying out the design of a working hydropower system especially at this era of need in the era of sustainable and dependable power generation.

Research Methodology

The research methodology consists in articulating a workable and simple application of fundamental mathematical relations aimed at arriving at acceptable and dependable results. As a result, a working flow chart was articulated by the researcher. This was meant to provide guidance as required. This flow chart is shown in Figure 1. Parameters whose mathematical models were deduced were the volumetric flow rate, (Q) water flow velocity, (V), length of penstock, (L), gate opening valve, (G), and power output, (P). Many fundamental mathematical principles were applied in arriving at these deduced formulae. This was to reduce the use of very complex mathematical computations in arriving at solutions which could have been otherwise achieved ordinarily. The research believes that applying these mathematical formulae in addition to adequate knowledge of the operation of the hydropower system, a good design could be achieved following the normal procedure.

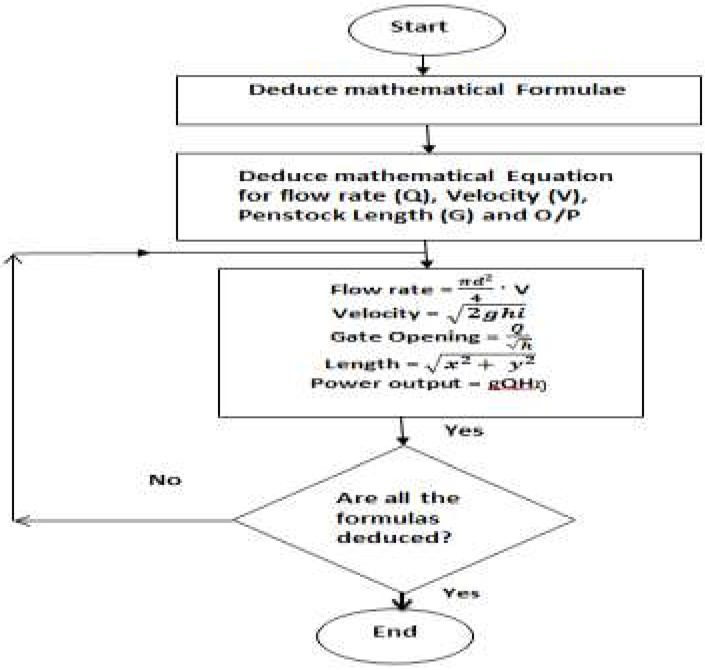
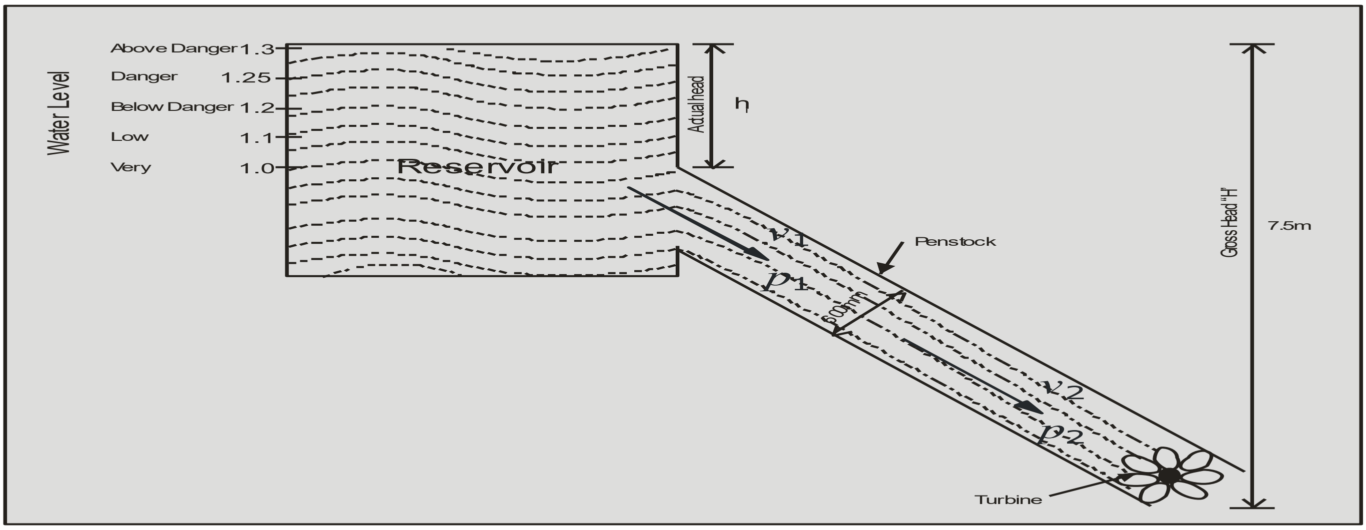


Figure 1: Flow Chart for the research realization

The flow chart for objective one is shown in Figure 1. It is meant in this presentation to deduce mathematical models, in the form of formulae, for the actual calculation and determination of the various parameters namely; Flow rate (Q), the Velocity (V), Penstock length, (L), Output power (P) and Gate opening (G). The deduced mathematical formulae for the variables are highlighted in this chapter as the report progressed. Following the same procedure, we calculated different values for velocity, flow rate, gate opening and output power, at different reservoir water heads of 1.1m, 1.2m, 1.25m, and 1.3m respectively. Following the same procedure, we calculated different values for velocity, flow rate, gate opening and output power, at different reservoir water heads of 1.1m, 1.2m, 1.25m, and 1.3m respectively. Figure 2 shows the arrangement for the flow of fluid /water from the reservoir through the penstock to the turbine.

The underlying aim of this presentation is to develop mathematical models and equations for use in calculating parametric values for hydropower system. Mathematics as an indispensable tool in scientific research and analysis lays claim to theoretical maturity, having developed a body of interrelated set of specific narrowly focused models. In view of this, the need to establish mathematical models and equations to be used in subsequent research works in solving given problems is treated in this paper.



Velocity of Fluid into the Penstock

Figure 2: Hydropower design Entry Arrangements: Velocity into the penstock

hi (actual head) is the distance between the water level of the reservoir and the gate opening of the penstock. H is the gross head, which is the vertical or perpendicular distance from the connection of the penstock to the gate opening on the reservoir to the turbine blade installation in the power generation assembly.

This arrangement enables us to calculate the Flow rate (Q), the Velocity (V), the Gate opening (G) and the Output power (P). The deduced mathematical formulae are applied as shown in the sample calculations. It is important to note that the reservoir was calibrated into different water levels, denoted as actual water heads hi, in the values of between 1m and 1.3m.

While 1.0 represents very low water, 1.3 represents above danger, with other actual water head values representing different safety conditions as shown. An expansion of the water flow through the penstock is shown in fig. 3.

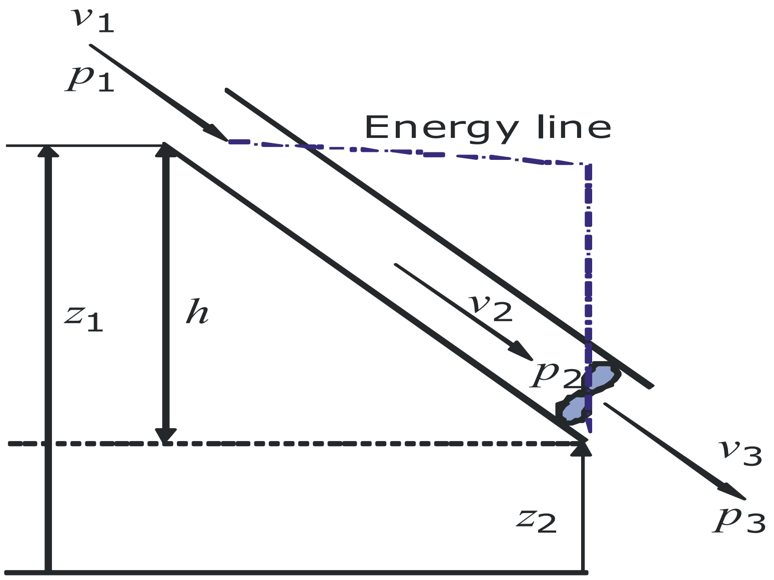


Figure3: Water velocity through the penstock to the turbine

Through the penstock, $V_1P_1 = V_2P_2 = V_3P_3$, meaning that along the penstock, water velocity, power transmission and power output are equal at any point within the diameter values. It should be recalled that in Figure 2, hi is the distance between the gate opening of the penstock and the water level in the reservoir while the pipe between the reservoir and the turbine is known as penstock. A typical value for penstock velocities with respect to a micro hydropower system varies from 2–5m/s.

Mathematical models are used by engineers to analyze systems to be controlled or optimized. In analysis, engineers establish mathematical models and equations to solve given problems in order to achieve expected design specifications.

To calculate the volumetric flow rate

Note that the volumetric flow rate of fluid is defined as the volume of fluid that is passing through a given cross sectional area per unit.

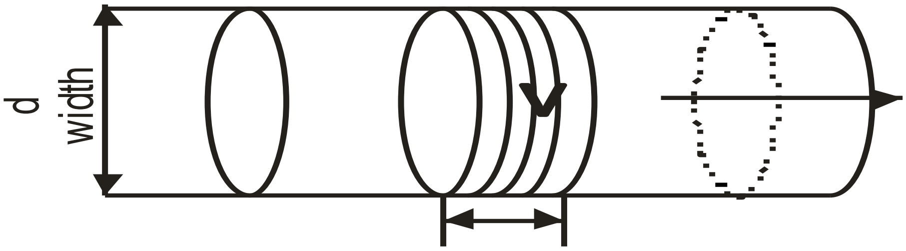


Figure 4: Volume of a portion of the fluid in a pipe

This is derived from;

$$\mathbf{Q} = \frac{\boldsymbol{V}}{\boldsymbol{t}}$$

This 'V' can be represented as; $V = Ad$

Where

A is the cross-sectional area of the fluid, and

d is the width of that portion of fluid.

Therefore; $Q = \frac{V}{t} = \frac{Ad}{t} = A\frac{d}{t}$

Hence the volume of a portion of fluid in a pipe can be represented as,

$$\mathbf{Q} = \mathbf{A}\frac{\boldsymbol{d}}{\boldsymbol{t}}$$

d/t is just the length of time the given volume of fluid is to flow through the specified unit length. This is the speed of the fluid. Replacing d/t with V in the previous equation, we will get a new equation, which is the formula for calculating the flow rate of water,

$$\mathbf{Q = AV}$$

where;

A is the cross-section area of the fluid in a pipe, given as = $\boldsymbol{\pi d^2} / \mathbf{4}$, and

V is the flow velocity.

Flow rate can be re-written as; $Q = \frac{\boldsymbol{\pi d^2}}{\mathbf{4}} * V$

The volume flow rate is measured in cubic meter per second, m^3/s.

To calculate the water velocity

To calculate the velocity of water flowing from the actual water head in the reservoir, through the gate opening into the penstock to the turbine blades, certain quantities and parameters are to be considered. These are the acceleration due to gravity, g, and the actual water head, hi.

Hence, for a micro hydropower system of a reservoir actual head (hi), considering the acceleration due to gravity g, the velocity is given as;

$$V = \sqrt{2ghi}$$

To calculate the length penstock

In calculating the length of the penstock, we take our gross water head as 7.5m

As shown in the Figure 1.

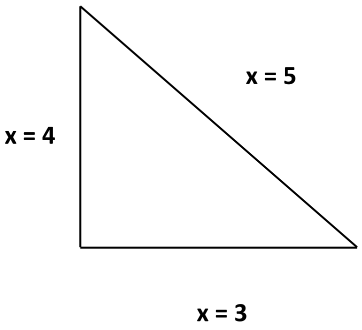


Figure 5: The Pythagoras theorem

To find the length of a penstock use the Pythagoras theorem; The Pythagoras theorem uses a standard right angled Δ of sides 3, 4, 5 as shown.

By relation,

$$z^2 = x^2 + y^2$$

so that,

$$z = \sqrt{x^2 + y^2}$$

Having known the measurement of the head, the length of the penstock can be calculated by using the basic right angled of the sides used in the Pythagoras theorem, any right-angled triangle can be completely solved.

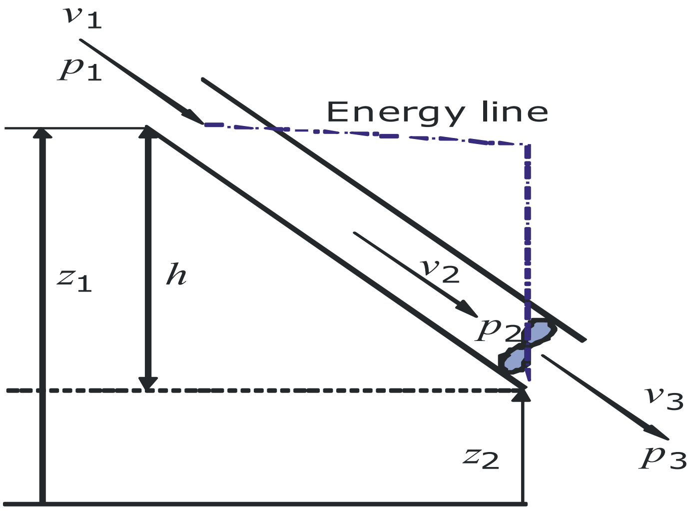


Figure 6: Flow of water in the penstock

Recall, that the basic equation of turbine penstock consists of the flow of water in penstock, mechanical power and acceleration of the water in the penstock. Hence, in fluid dynamics, the volume of fluid which passes per unit time is usually represented by the symbol Q (sometimes V): The motion of water in the penstock can be modified as shown in equation 3.7;

$$\theta = G\sqrt{H}$$

$$Q = (f, \text{gate \& Head})$$

$$Q = f, (GH)$$

Where θ is the flow rate in m^3/s

G is gate opening in rad.

H is gross head in meter.

Note that the velocity of water in penstock varies directly with the gate opening.

To calculate the power generated

Recall that the actual power generated from the given source of water as shown in chapter equation (1) is reproduced as equation

$$P = gQH\eta$$

This means that to calculate the generated output power, knowledge of the volumetric water flow rate as well as that of the overall water head, (H) was needed. Also, the constant, η, which is a measure of the efficiency of the generator performance, should be added in the multiplier.

Discussion

Mathematical models are meant to help in explaining the functions and behavior of a system and to study the effects of its different components thus making dependable predictions about the system's behavior. The evaluation of whether a given mathematical model describes a system accurately or not is a major consideration in a mathematical modeling process. In model evaluation, it is usually advised that a check be made on whether it fits experimental measurements or other empirical data. According to how much a priori information is available of a system, mathematical modeling problems are classified into black box or white box models. When there is no a priori information available in a system, it is known as a black box model. Conversely, where all necessary information is available in a system, it is known as a white box (also known as glass box or clear box). However, to make a model more accurate, as much a priori information as possible should be used. This was the case in the presentation made in this paper. As much as desirable and possible, relevant priori information is given in order to arrive at deductions made. It is hoped that the formulae would be handy in future researches in the hydropower system technology.

Conclusion

This paper presented, in a very simple but specific procedure, some fundamental formulae for the calculation of relevant parameters used in hydropower design and implementation. It is devoid of complicated and lengthy mathematical derivations and proofs which may require mastery in their use and application to arrive at the expected and desired results. In the next paper, the result of an application of these deduced mathematical models in the design of a hydropower system will be presented. The popularity of the deduced models is expected to receive a wide acceptance in subsequent designs and construction of hydropower systems especially as the need for the development of small power generation units continues to increase

References

G. A. Leonov, et al; A Simple Dynamical Model of Hydropower Plant: Stability and Oscillations, IFAC-Papers Online, Volume 48, Issue 11, 2015.

Runfan Zhang, et al; Nonlinear Predictive Control of a Hydropower System Model, Entropy Journal, 1 September, 2015.

Caoyuan Ma, et al; Fixed-Time Stability of the Hydraulic Turbine Governing System, Mathematical Problems in Engineering, Volume 2018, 10 pages.

Alexander Zaretskiy; Mathematical Models and Stability Analysis of Three-Phase Synchronous Machines, Jyvaskyla Studies in Computing, 179, 2013.

Weijia Yang, et al; A Mathematical Model and its Application for Hydropower Units under Different Operating Conditions, Open Access Energies, 2015, 8, 10260 – 10275.

Sachin Mishra, et al; Effect of Variation of Penstock Parameter on Mechanical Power, International Journal of Energy Science, 2012. Vol. 2, Issue 3, pp110-114.

Amit Anakiya, et al; Analysis of the Penstock for Pit Turbine using Computational Fluid Dynamics, IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) e-ISSN: 2278-1684, p-ISSN: 2320-334X, Volume 11, Issue 3 Ver. V (May- Jun. 2014), PP 36-42.

Arun Kumar and Mukesh Kumar Singhal; Optimum Design of Penstock for Hydro Projects, ResearchGate, August 2015