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RESEARCH ARTICLE

Kinematic and Dynamitic of Robotic Arm Using Four Degrees of Movement

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ABSTRACT

This research paper used the kinematic and dynamitic of robotic arm to demonstrate the movement of a robot arm. Two mathematical models were considered and developed. The first is to solve the problem of accuracy (position) and the second is to solve the problem of speed. For the accuracy or position we consider the kinematics of the robot arm which will show the movement of a robot to a particular position without missing its target. The accuracy at this point is the ability of the robot to place itself to a desired point or particular location with the minimal error. The second model considers the speed of movement, that is the distance covered per unit time, or the rate at which the robot can move usually in m/s. For the speed of the robot, a DC motor which usually moves the robot must be considered in the modelling. The following objectives was used to achieved the aim of this paper: develop mathematical model for robot arm, determine four degrees of freedom of robot arm, determine and develop angles and distance of movement with respect to degrees of freedom, develop mathematical model for the speed of robot, generate the voltage and speed of robot arm and develop the entire models of robot arm position and speed

Keywords: Robotic Arm, Kinematic of Robot Arm, Dynamitic of Robot Arm, Mathematical Model

Introduction

The movement of robots is caused by direct current (DC) motors. The components of DC motors consist of electromechanical systems and that is the major reason for its wide applications in industries. DC motor is an inherently high speed and low torque device which changes electrical energy to mechanical energy (S. M. Labaran and H. A. Bashir 2018). Most mechanical systems in industries are composed of masses moving under the action of the position and velocity-dependent forces (Nayana P, Mahajan, S.B. Deshpande, 2013). Servo DC motors are suitable for robotics arm because it has decent torque and precise control over angular rotation (Fariz Ali, Junior Sintar, M.A, A.Z, 2019). It can also be controlled by using pulse width modulation (PWM) through the control wires in some applications. Robots have some body parts which it uses to work, which can be referred to as their own anatomy or body make-up. The anatomy is made up of the physical construction of the body which consists of the arm and wrist of the machine which represents the robot. For the purpose of this paper, the robotic arm movement will be discussed. The arm of a robot is used to perform its duties just like a human arm, such duty as moving one thing from one location to another. The arm of a robot is just like the one of a human being, this is because it has a joint that permits it to have freedom of movement and to any degree. The robotic arms cannot be discussed without knowing the Degrees of Freedom (DOF). This can be defined as the number of variables required for the motion of a body in space, which may be of a maximum number of six. (Khushdeep G and Deepak B. 2011). Each joint in a robot represent a degree of freedom (DOF) which is the movement involved and each degree of freedom

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(DOF) can be a slider, bend, rotation or other types of actuator for movement. The actuator permits the movement or motion of the robot at that particular joint. Robots have many degrees of freedom depending on the function it has to perform. Most often robotic arm has between one to six degrees of freedom, 3 of the degree of freedom are used for the positioning in 3D space while the other 3 are used for orientation of the end effectors. Most often, six (6) degrees of freedom are enough for the robot to reach all positions and orientations. According to Khushdeep G and Deepark B, a joint in a robot is where points or links are connected and these connections allow motion or movement of the robot and the DOF can be a rotational or translational degree of freedom. (Khushdep Goyal and Deepark Bhandam 2011)

Furthermore, a robot has two major types of joints, which are the Revolute and lever point. The revolute joints are normally driven by electric motors and chain /belt /gear transmissions or by hydraulic cylinders, while the levers joints are the Prismatic, which uses the slider joints to link to other parts. The lever joint supports the linear slider bearing and linearly actuated the ball screw, motors or cylinders. (Khushdep Goyal and Deepark Bhandam 2011)

The Dynamics and Kinematics of Robotic Arm

Research shows that the dynamics of a robotic arm is mostly derived from the Lagrange-Euler formulation. To explain the problems involved in dynamic modelling, figure 1.0 is used. Figure 1.0 shows the diagram of a two degree of freedom (DOF) of the robotic arm. The robot arm has link1 and link 2 with joint displacement that links the lengths. The joint displacement also represents the masses of each link and torque for links 1and 2 respectively. Before elucidating the model, the following assumptions were made: i. The actuators dynamics for motor and gearboxes will not be considered. ii. The effect of friction forces will be taken to be negligible iii. The mass of each link is taken to be concentrated at each end link.

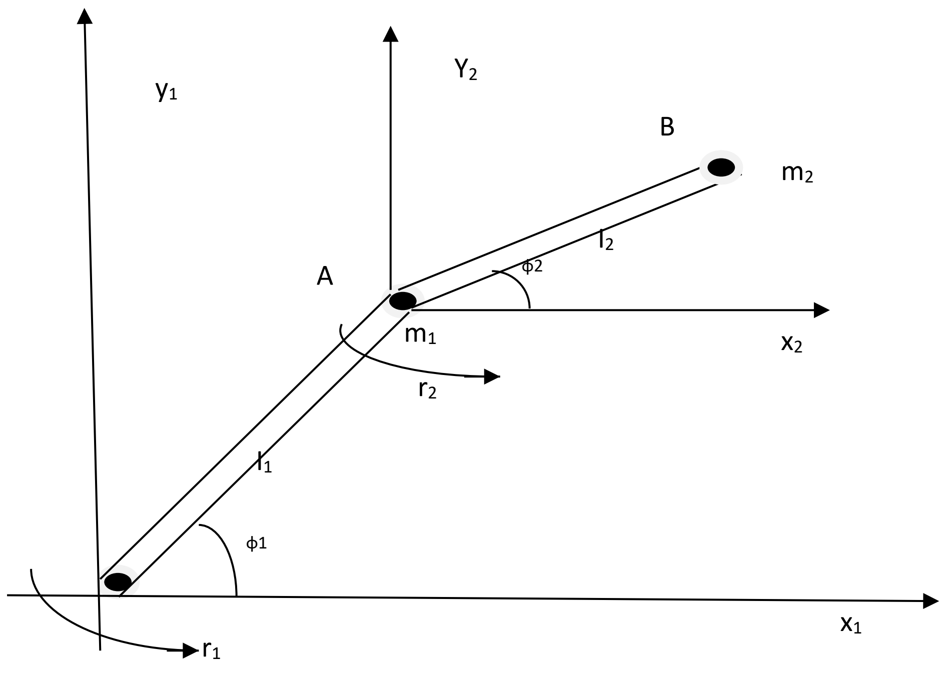

Figure 1: Representation of Robot Arm with 2- DOF

The energy in the system has to be considered, and the two energies to be calculated are kinetic and potential energies of the system. Therefore, the kinetic energy of the manipulator or robotic arm as a function of joint position and velocity can be expressed in equation (1):

$$k(\theta\ \vartheta) \quad = 1 + \frac{1}{2}\theta tm\ (\theta)\vartheta(+\frac{n(n-1)MV^2}{2} \qquad (1)$$

Where ($M\theta$) is the nxn manipulator mass matrix and the subscript i denote 1and 2. Hence, the total kinetic energy of the robot arm is the sum of the kinetic energies and of the individual link as shown in figure 1

The degree of freedom (DOF) of an arm of a robot is the number of independent variables required to show the location of all parts of the mechanism, that is the number of points where there is a joint in the system or the robot arm. (Khushdeep G and Deepak B. 2011) An industrial robot is an open kinematic chain that usually the number of joints is normally the same as the number of degrees of freedom DOF. Freedom of movement is only experienced at the joint of a robot. Therefore, the degree of freedom happens when there is a joint or connection. According to khushdeep G and Deepak B, 2007, robot arm kinematics is all about the study of the direction of motion of a robot arm with regards to a fixed reference coordinate system as a function of time without regard to the forces/moments which causes the motion of the robot. In other words, the degree of freedom can be defined as the detailed analysis describing the displacement of the robot in respect of time in particular and the relations between the joint variable, the position and the end effectors of a robot arm. The maximum DOF required to position end effectors in space is six and they relate to position: x, y,z coordinates and orientation. (Khushdep Goyal and Deepark Bhandam, 2011) Kinematics of robots can also be expressed by using four quantities. Two of them are coming from the link itself and the rest comes from the relations between links neighbours. (Kaan Yilmaz Ekrem Berkmen Boray Yurdakul, 2017)

Development of a Mathematical Model

The mathematical model of the robot arm was developed. The flow chart and algorithm are shown below, in figure 2

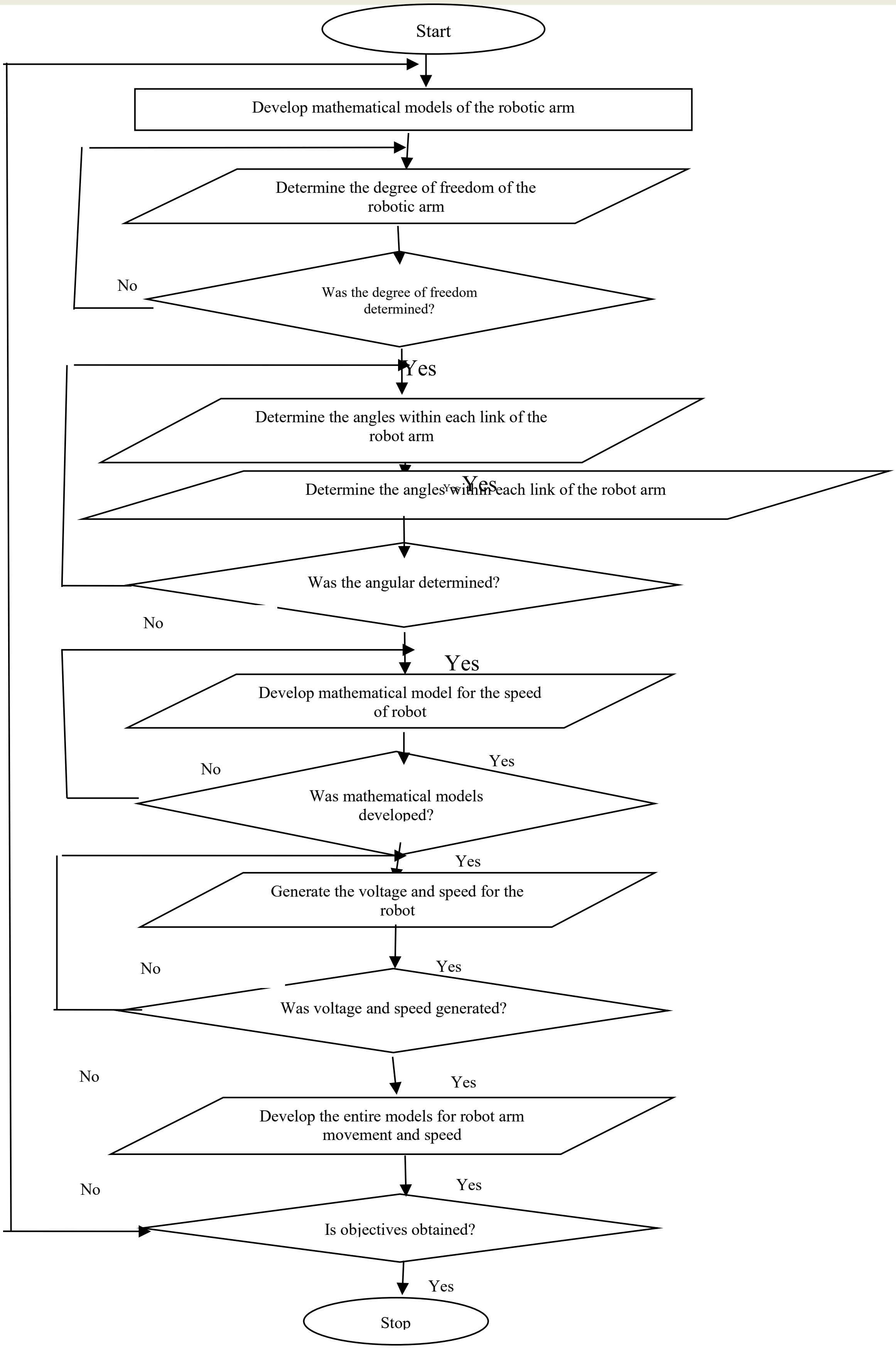

Figure 2: Flow Chart for a Developed Mathematical Model of the Robot Arm

In this research paper, two mathematical models were considered and developed. The first model was to solve the problem of accuracy (position) and the second was to solve the problem of speed. For the accuracy or position, we consider the kinematics of the robot arm which will show the movement of a robot to a particular position without missing its target. The accuracy at this point is the ability of the robot to place itself to the desired point or particular location with minimal error. The second model considers the speed of movement, that is the distance covered per unit time or the rate at which the robot can move usually in m/s. For the speed of the robot, a DC motor that usually moves the robot must be considered in the modelling.

Mathematical Model of 4 Degree of Freedom (DOF) Robotic Arm

A mathematical description of a robot arm can be shown using position and joint angles. To develop a mathematical model for the robot arm, the kinematics and dynamics of the robot arm must be considered and this can be done using degree of freedom (DOF). The function describing the position and the joint angle is known as the forward Kinematics function. The forward kinematics analysis is done for the flexibility of each of the joint angles which normally shows the position and orientation of the end effectors. (D. Sivasong, M. Dev Anand 2019). In this research, we considered 4 degrees of freedom which will enable the robot to pack crates of soft drinks from the conveyor and drop it to the pallet. The dynamics and the kinematics of the robot arm are shown in the mathematical model below. The schematic diagram of a four-degree of freedom (DOF) of the robot is shown in figure 3 below.

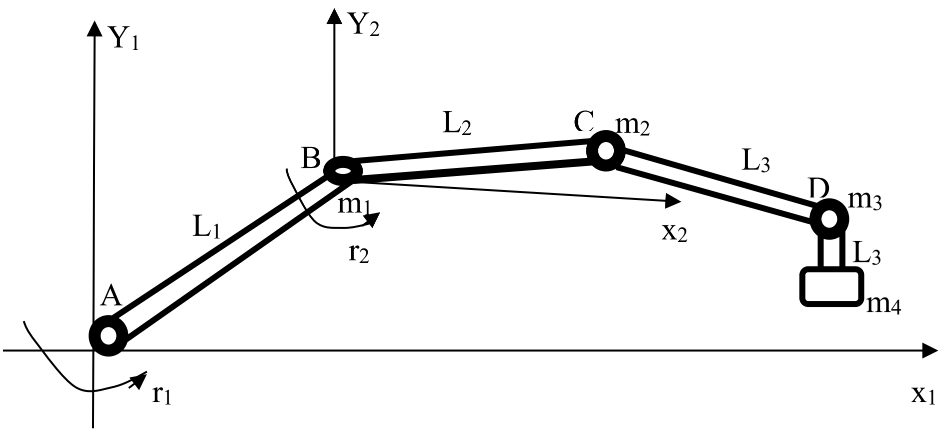

Figure 3: Robotic Arm with 4 Degree of freedom (DOF)

Figure 3 consist of a robot arm with link 1, 2, 3 and 4, joint displacement of θ_1, θ_2, θ_3 and θ_4, link length L_1, L_2, L_3 and L_4, M_1, M_2, M_3 and M_4 which are the masses for link 1, 2, 3 and 4 respectively. while $Ɩ_1$, $Ɩ_2$, $Ɩ_3$ and $Ɩ_4$ are the torque for all the links, that is link 1, 2, 3 and link 4 respectively. Three basic assumptions are made in the model, which is as follows: the arms dynamics of motor and gearboxes are not taken into account, friction forces effects are assumed to be negligible and the mass of each link is assumed to be concentrated at the end of each link. The kinetic energy of the arm as a function of joint position and velocity is expressed in equation (1) and the kinetic energy and the potential energy of the system should be calculated

$$k(\theta\ \vartheta) = \frac{1}{2} M\theta' Ɩm\ (\theta)\vartheta = \frac{M1V^2}{2} \qquad (1)$$

Where, $(M\theta)$ is the nxn manipulator mass matrix and the subscription I denote 1 and 2, thus the total kinetic energy of the system (robot) is the sum of the individual kinetic energy of the two links termed K_1 for link1 and K_2 for link 2.

$$k(\theta\ \vartheta) = \sum Ki(\theta, \vartheta) \qquad (2)$$

$$k(\theta\ \vartheta) = \frac{M1V1^2}{2} + \frac{M2V2^2}{2} + \frac{M3V3^2}{2} + \frac{M4V4^2}{2} \qquad (3)$$

To calculate K_1 and K_2, the position equation for m^1 at A and m^2 at B is differentiated using a linear product to obtain their respective velocity

$$x1 = L1sin\,\theta1 \quad (4)$$

$$y1 = L1cos\,\theta1 - \quad (5)$$

$$x2 = L1sin\,\theta1 + L2sin\,(\theta1 + \theta2) \quad (6)$$

$$y2 = -L1cos\,\theta1 - L2cos\,(\theta1 + \theta2) \quad (7)$$

$$x3 = L1sin\,\theta1 + L2sin\,(\theta1 + \theta2) + L3sin\,(\theta1 + \theta2 + \theta3) \quad (8)$$

$$y3 = L1cos\,\theta1 + L2cos\,(\theta1 + \theta2) + L3cos\,(\theta1 + \theta2 + \theta3) \quad (9)$$

$$x4 = L1sin\,\theta1 + L2sin\,(\theta1 + \theta2) + L3sin\,(\theta1 + \theta2 + \theta3) + L4sin\,(\theta1 + \theta2 + \theta3 + \theta4) \quad (10)$$

$$y4 = -L1cos\,\theta1 - L2cos\,(\theta1 + \theta2) - L3cos\,(\theta1 + \theta2 + \theta3) - L4sin\,(\theta1 + \theta2 + \theta3 + \theta4) \quad (11)$$

Development of the Angles and Distance of Movement

The control strategy here is to control each of the links effectively and independently, unfortunately, there is a strong interaction between the four links. The coupling effect needs to be decoupled to gain enough freedom to control each link freely. The link and joint movement are used to control the robot arm to move and stop at a particular position that is to pick the crate and drop it at a particular position. From the figure below, it is quite obvious that while link 1 is turning horizontally link 2, 3 and 4 turns vertically. Figure 4 shows that link 1 which is the horizontal link is the base.

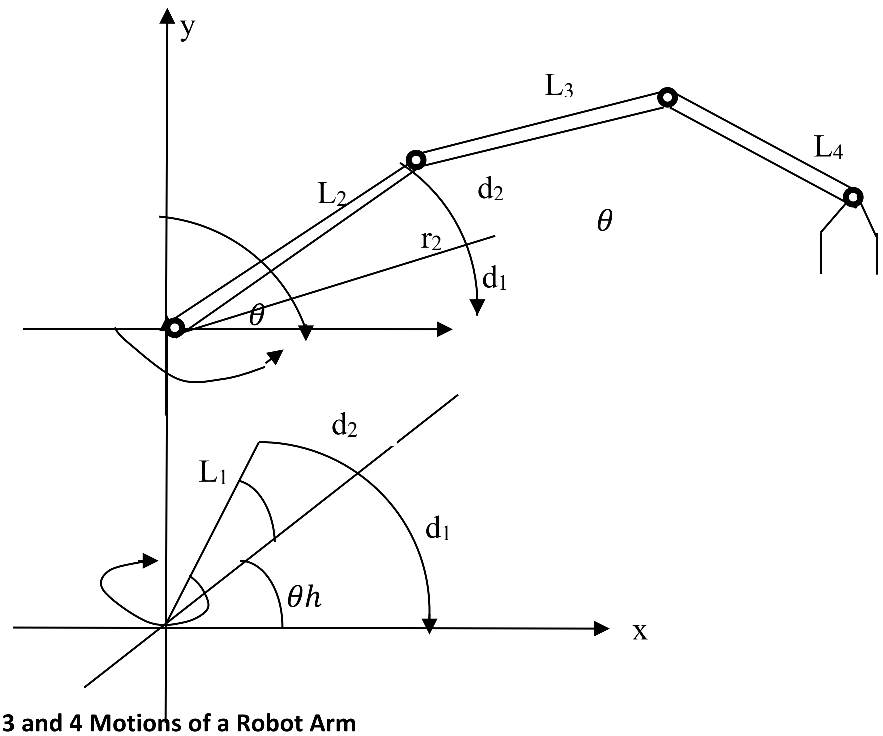

Figure 4: Link 1, 2, 3 and 4 Motions of a Robot Arm

From the diagram in figure 4.0, it is quite obvious that the true position of any of the links is determined by the arc formed by each arm (horizontal for link 1 and vertical for links 2, 3 and 4). This arc is determined by the $\theta H\ or\ \theta V$ is suspended by the links L_1, L_2, L_3 and L_4 respectively above their axes. To achieve the desired set (point) joint angle θd, robot control should design input angle θ such as the regulation errors.

Therefore $$\dot{O} = \theta d - \theta \qquad (11)$$

Where,

θd is the desired joint, θ - is the actual joint angle and $\dot{O}$ - is the angle error. When this is maintained, the links will always be at the correct position. The tables below were the angles (θ) with the corresponding distance for the movement of link 1, link 2, link 3 and link 4.

$$d = \frac{\theta}{360} X \frac{2\pi R}{1} \qquad (12)$$

Where θ the angle subtended by each link or arm

R the length of the link

d distance or angular displacement by the link

When the input is θ then

$$\theta = \frac{360\ X\ d}{2\pi R} \qquad (13)$$

With equations 12 and 13 we can calculate the value of distance and angle respectively for all the links, and this is shown in the tables below. The multiple data acquired as distance and angles which will be shown in tables below were generated to overcome the limitation placed by the boundary conditions.

Horizontal Arm Link 1 with 80 Degrees Maximum

This arm has maximum degrees of 80 and the corresponding distance for the degrees was generated, which gave it the advantage of having almost no boundary condition, with this the robot arm can be used in many other places to perform other functions since it can be adjusted to different angles and distance.

Table 1: Horizontal Link 1 of 4 DOF Robot Arm

θ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50
d	0.39	0.13	0.17	0.22	0.26	0.30	0.34	0.39	0.43	0.47	0.52	0.56	0.60	0.65	0.69	0.73
θ	9.00	9.50	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5
d	0.76	0.82	0.86	0.90	0.94	0.99	1.03	1.08	1.11	1.16	1.21	1.25	1.29	1.33	1.38	1.42
θ	17.0	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23.0	23.5	24.0	24.5	25.0	25.5	26.0
d	0.78	1.64	1.68	1.72	1.77	1.81	1.85	1.89	1.94	1.98	2.02	2.07	2.11	2.15	2.20	2.24
θ	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0	31.5	32.0	32.5	33.0	33.5	34.0
d	2.28	2.33	2.37	2.41	2.45	2.50	2.54	2.58	2.63	2.67	2.71	2.76	2.80	2.84	2.88	2.93
θ	34.5	35.0	38.0	39.0	40.0	41.0	41.5	42.0	42.5	43.0	43.5	44.0	44.5	45.0	45.5	46.0
d	2.97	3.01	3.30	3.40	3.44	3.53	3.57	3.62	3.66	3.70	3.75	3.79	3.83	3.88	3.92	3.96
θ	46.5	47.0	47.5	48.0	48.5	49.0	49.50	50.0	50.5	51.0	51.5	52.0	52.5	53.0	53.5	54.0
d	4.00	4.05	4.09	4.13	4.18	4.22	4.26	4.31	4.35	4.39	4.44	4.48	4.52	4.56	4.61	4.65
θ	54.5	55.0	55.5	56.0	56.5	57.0	57.5	58.0	58.5	59.00	59.5	60.0	60.5	61.0	61.5	62.0
d	4.49	4.74	4.78	4.82	4.87	4.91	4.95	4.99	5.04	5.08	5.12	5.17	5.21	5.25	5.29	5.33
θ	62.5	63.0	63.5	64.0	64.5	65.0	65.5	66.0	66.5	67.0	67.5	68.00	68.5	69.0	69.5	70.0
d	5.38	5.43	5.47	5.51	5.55	5.60	5.64	5.68	5.73	5.77	5.81	5.86	5.90	5.94	5.98	6.03
θ	72.5	73.0	73.5	74.0	74.5	75.0	75.5	76.0	76.5	77.0	77.5	78.0	78.5	79.0	79.5	80.0
d	6.24	6.29	6.33	6.37	6.42	6.45	6.50	6.54	6.59	6.63	6.67	6.71	6.76	6.80	6.85	6.88

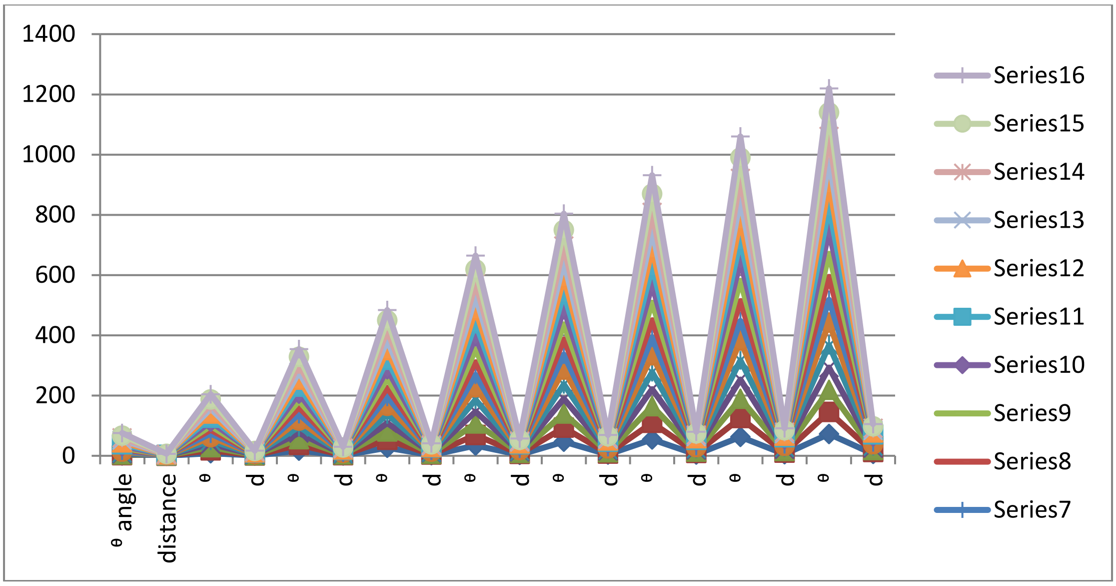

Figure 5: Angles and distance of horizontal arm link 1

The graph shows that the horizontal movement can cover from 1 degree to 80 degrees with the corresponding distances, as shown in figure 5. A wide range of the degree of movement is used at the base to cover the range of other links, this is because the base which is the horizontal moves with other links which are the vertical links.

Vertical Arm with 45 Degrees Maximum for Link 2

Link two is the first vertical link with 45 degrees maximum angle. It equally has many angles and distances for adjusting the positions. The angles with the corresponding distance are shown in table 2 below

Table 2: Vertical Arm Link 2 of 4 DOF Robot Arm

θ	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	7.0	7.5	8.00	8.50	9.0
d	0.09	0.13	0.17	0.22	0.26	0.30	0.34	0.39	0.43	0.47	0.52	0.60	0.65	0.69	0.73	0.78
θ	9.50	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0
d	0.82	0.86	0.90	0.95	0.99	1.03	1.08	1.12	1.16	1.21	1.25	1.29	1.33	1.38	1.42	1.46
θ	19.0	19.5	20.0	20.50	21.0	21.5	22.0	22.5	23.0	23.5	24.0	24.5	25.0	25.5	26.0	26.5
d	1.64	1.68	1.72	1.77	1.81	1.85	1.89	1.94	1.98	2.02	2.07	2.11	2.15	2.20	2.24	2.28
θ	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0	31.5	32.0	32.5	33.0	33.5	34.0	34.5
d	2.33	2.37	2.41	2.45	2.50	2.54	2.58	2.63	2.67	2.71	2.76	2.80	2.84	2.88	2.93	2.97
θ	35.0	38.0	38.5	40.0	41.5	43.0	44.5	45.0								
d	3.01	3.23	3.31	3.44	3.57	3.70	3.83	3.88								

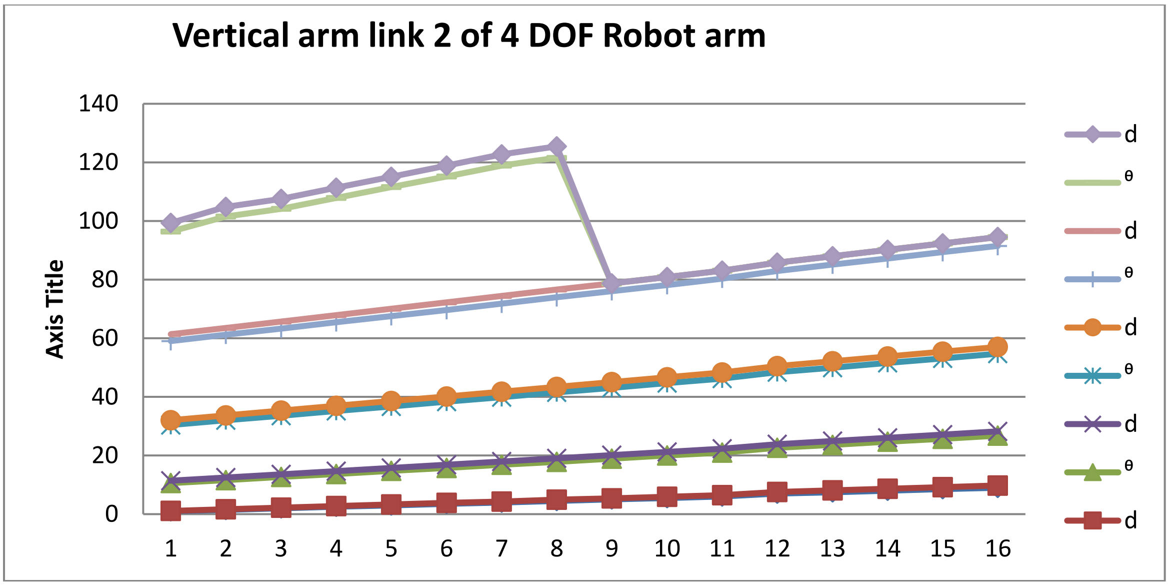

Figure 6: Angles and Distance of Vertical Arm Link 2 of 4 DOF Robot Arm

The graph shows that the horizontal movement can cover from 1 degree to 45 degrees with the corresponding distances, as shown in table 2 and figure 6. it shows the level of degree of movement within this link.

Vertical Arm with 30 Degrees Maximum for Link 3

Link 3 is the second vertical link with a maximum angle of 30 degrees. From the data generated for the link, it was seen that it equally has many angles and distances for multiple positions

Table 3: Vertical Arm Link 3 of 4 DOF Robot Arm

θ	0.00	0.5	0.1	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5
D	0.00	0,03	0,07	0.10	0.14	0.17	0.21	0.24	0.28	0.31	0.35	0.38	0.42	0.45	0.49	0.52
θ	8.00	8.50	9.00	9.50	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5
D	0.56	0.59	0.63	0.66	0.70	0.73	0.77	0.80	0.84	0.87	0.91	0.94	0.98	1.01	1.05	1.08
θ	16.0	16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23.0	23.5
D	1.12	1.15	1.19	1.22	1.26	1.29	1.33	1.36	1.40	1.43	1.47	1.50	1.54	1.57	1.61	1.64
θ	24.0	24.5	25.0	25.5	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0			
D	1.68	1.71	1.75	1.78	1.82	1.85	1.88	1.92	1.95	1.99	2.02	2.06	2.09			

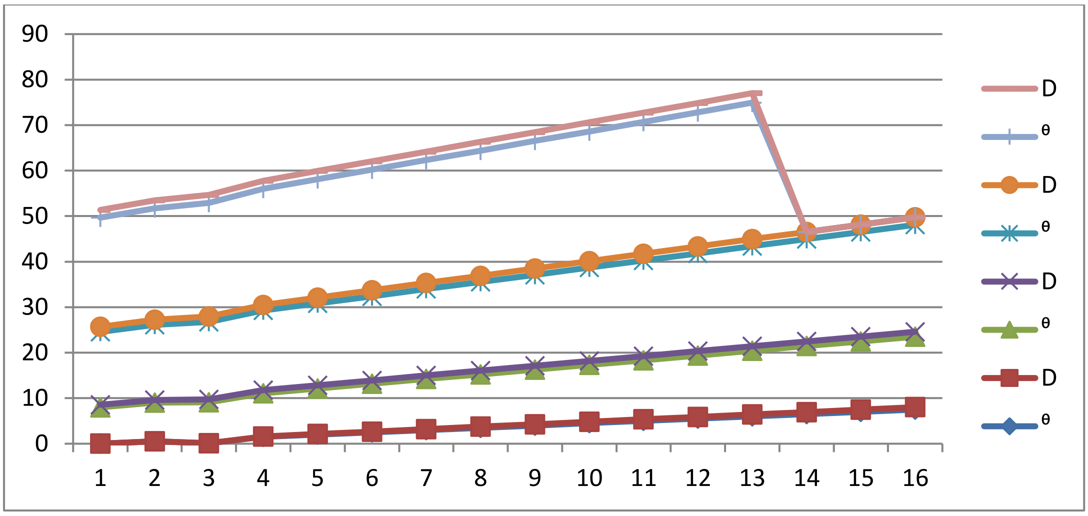

Figure 7: Angles and distance of Vertical arm link 3 of 4 DOF Robot arm

The graph of figure 7 shows the vertical movement which covers from 1 degree to 30 degrees with the corresponding distances, as shown in table 3. It shows the level of degree of movement within link 3.

Vertical Arm with 30 Degrees Maximum for Link 4

Link 4 is the third and last vertical link with a maximum angle of 30 degrees also. From the data generated for the link, it was seen that it equally has many angles and distances for multiple positions. The angles and the corresponding distance were calculated and shown in table 4 below. It was also noted that each of the links can only move one after the other

Table 4: Vertical Arm Link 4 of 4 DOF Robot Arm

θ	0.0	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.0	4.50	5.00	5.50	6.00	6.50	7.00	7.50
D	0.0	0.03	0.05	0.08	0.10	0.13	0.16	0.18	0.21	0.24	0.26	0.29	0.31	0.34	0.37	0.39
θ	8.00	8.50	9.00	9.50	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5
D	0.42	0.45	0.47	0.50	0.52	0.55	0.58	0.60	0.63	0.65	0.68	0.71	0.73	0.76	0.79	0.81
θ	16.0	16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23.0	23.5
D	0.84	0.86	0.89	0.92	0.94	0.97	0.99	1.02	1.05	1.07	1.10	1.13	1.15	1.18	1.20	1.23
θ	24.0	24.5	25.0	25.5	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0			
D	1.26	1.28	1.31	1.34	1.36	1.39	1.41	1.44	1.47	1.49	1.52	1.54	1.57			

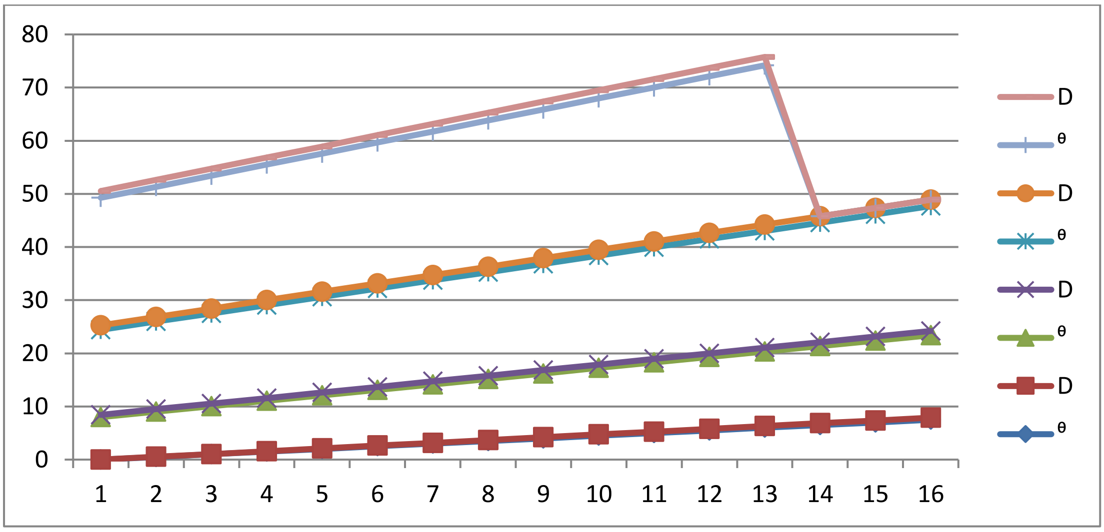

Figure 8: Angles and distance of Vertical arm link 4 of 4 DOF Robot arm

The graph shows that the vertical movement can cover from 1 degree to 30 degrees with the corresponding distances, as shown in table 4. It shows the level of degree of movement within link 4.

Mathematical Model for the Speed of Robotic Arm

The speed of the robot has to do with the DC servo motor. This is because the movement of a robot is caused by a direct current motor and the circuit diagram has to be considered. In a DC motor, the stator provides a rotating magnetic field that drives the armature to rotate. A simple DC motor uses a stationary set of magnets in the stator, and a coil of wire with a current running through it to generate an electromagnetic field aligned with the centre of the coil. The direct current motor is an important drive configuration for many applications across a wide range of power and speed; it has variable characteristics and is used extensively in variable speed drives (Nurshahirah Shaharudin, Mohd Z. H and Syatirah M. N 2021). Thus, the speed of a DC motor can control in many ways: by varying the flux, by varying the current through field winding, by varying the armature voltage, and by the armature resistance.

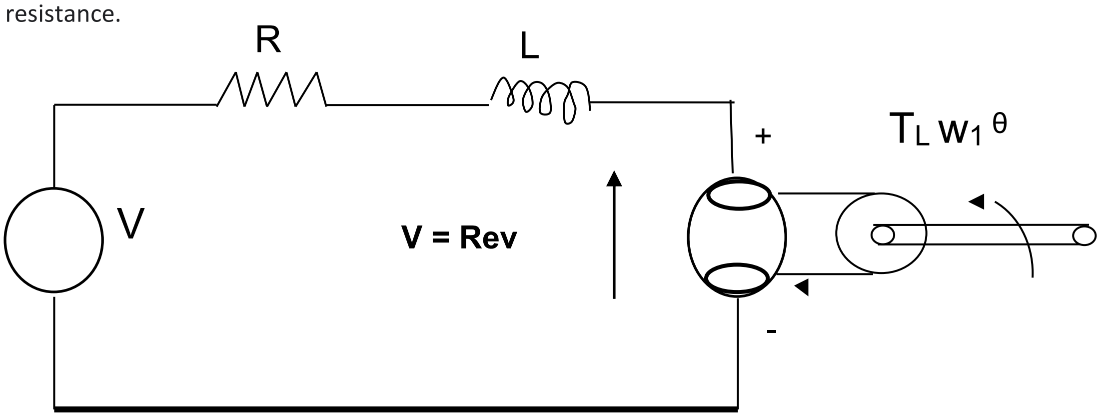

Figure 9: Servo Motor Circuit Diagram

The schematic diagram of the dc servomotor system is shown in figure 9. The input is the armature voltage (V) in volts being driven by a voltage source. The measured variables are the angular velocity of the shaft ω in rpm and the input voltage in volts. The system equations are shown in equations (1), (2), (3) and (4), which are derived from the diagram of figure 9.0.

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + emf_m \quad (14)$$

$$e_m = K_e \omega_m \quad (15)$$

$$T_m = K_t i_a \quad (16)$$

$$T_m = I_m \omega_m + b\omega_m + T_L \quad (17)$$

Where R is equivalent motor resistance, L is equivalent motor inductance, V is applied voltage,

emf_m is motor back emf, K_i is motor torque constant, J_m is the equivalent moment of inertia reflected at the motor shaft, b_m is the equivalent viscous coefficient reflected at the motor shaft, i_a is the armature current, K_e is the motor voltage constant, T_m is torque generated by the motor, ω_m is the motor speed, T_L is the load torque, Motor torque, T is related to the armature current I by a constant factor k, $T = ki$ (18)

Back electromotive force emf (Vb) is related to the angular velocity by

$$V_b = k\omega = k\frac{d\theta}{dt} \quad (19)$$

Using Newton's laws and Kirchhoff's laws, the following equations are generated from figure 9.0.

$$J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} = ki \quad (20)$$

$$L\frac{di}{dt} + Ri = V - k\frac{d\theta}{dt} \quad (21)$$

Transfer Function

Applying Laplace transform to equation 20 and 21 results in equation 22 and 23 below.

$$Js^2\theta(s) + bs\theta(s) = kI(s) \quad (22)$$

$$LsI(s) + RI(s) = V(s) - ks\theta(s) \quad (23)$$

Where s denotes the Laplace operator,

From equation 3.13

$$I(s) = \frac{V(s) - ks\theta(s)}{R + Ls} \quad (24)$$

and substituting this in equation 22 results in

$$Js^2 + bs\theta(s) = k\frac{V(s) - ks\theta(s)}{R + Ls} \quad (25)$$

Expanding and re-arranging of equations 25 yields the transfer function from input voltage, V(s), to the output angle Ɵ in equation 26

$$Ga(s) = \frac{\theta(s)}{V(s)} = \frac{k}{s\left[(R + Ls)(Js + b) + k^2\right]} \quad (26)$$

Since $\omega_m = \dot{\theta}$ (27)

Laplacing $\omega(s) = s\theta(s)$ (28)

Substituting $\theta(s)$ in equation 26 yields the transfer function of input voltage, V(s) to the output angular velocity, ω as

$$Gv(s) = \frac{\omega(s)}{V(s)} = \frac{k}{(R + Ls)(Js + b) + k^2} \quad (29)$$

The equations for the dc motor are shown in the block diagram in figure 10 below.

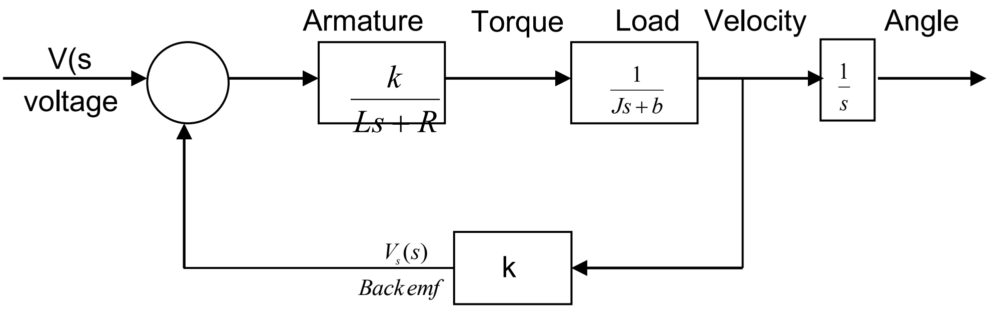

Figure 10: Block diagram of DC Motor Mathematical Model for the Speed of Robot Arm

This can be shown when the actual parameters from the DC series motor were used to do the simulation. In the implementation, the following actual motor physical parameters below are used.

Moment of inertia of motor J= 0.0062kg-m^3, Damping coefficient b = 0.001Nms/rad, Torque constant K_t = 0.06Nm/A, Electric resistance R = 2.2Ω, Electric Inductance L = 0.5H, Electromotive force constant K_e = 0.06Vs/rad. The figure above shows the development of the entire models of robot arm position and speed. The mathematical model of the DC motor of figure ---- can be used to control the position and speed of a robotic arm, and the multiple data generated as distance and angles can also be used to position the robot to do multiple works. These values are substituted for the parameter values in figure 3.13 above to yield the Simulink block diagrams below

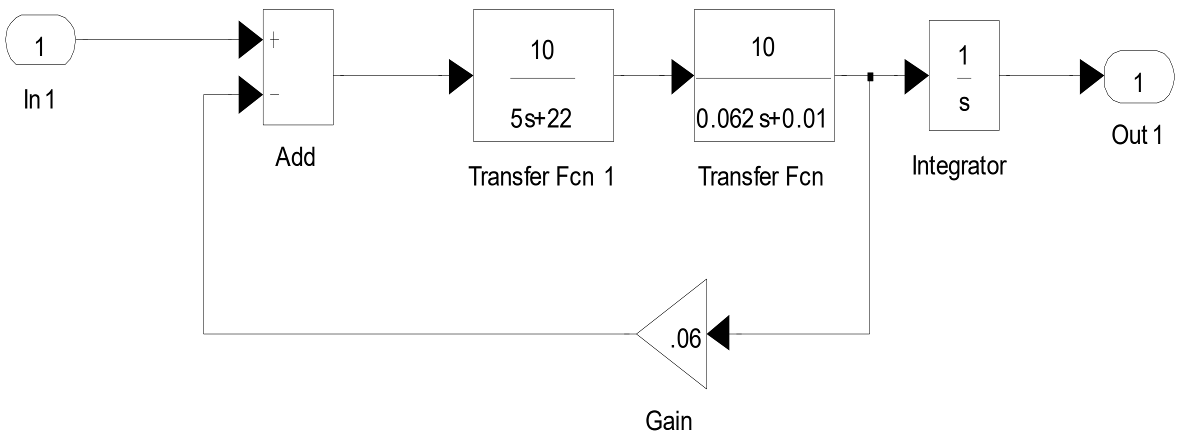

Figure 11: Block diagram of the model of the robot arm for speed control.

Table 5: Generated Speed from the Designed Model when Voltage is used as the Input

S/N	*Measured Voltage (Volts)*	*Speed from the Designed Model (m/s)*
1	0.90	202.1
2	1.22	273.9
3	1.28	287.4
4	1.30	291.9
5	1.33	298.6
6	1.36	305.3
7	1.50	336.8
8	1.60	359.2
9	1.70	381.7
10	1.80	404.1
11	2.20	493.9
12	2.40	538.8
13	2.60	583.7

14	3.20	718.4
15	3.40	763.3
16	3.65	819.4
17	4.00	898.0
18	4.21	945.1
19	4.80	1078.0
20	5.00	1123.0
21	5.80	1302.0
22	6.20	1392.0
23	6.80	1527.0
24	7.00	1572.0
25	7.50	1684.0
26	8.20	1841.0
27	8.80	1976.0
28	9.50	2133.0
29	10.00	2245.0
30	10.5	2357.0

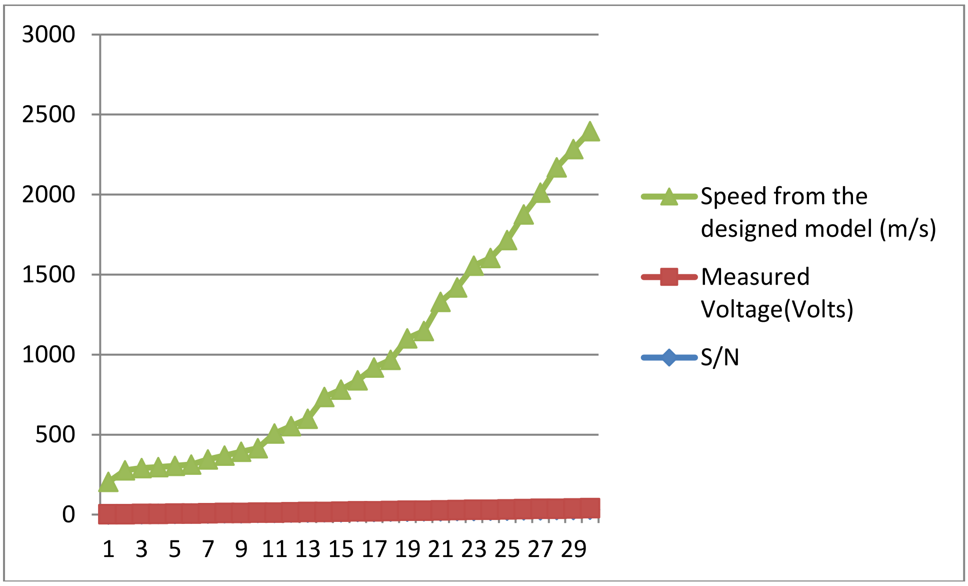

Figure 12: Speed movement with input voltage

Figure 12 shows the speed of the robot with input voltage. A mathematical model has also proved that an increase in input voltage increases the speed of the robot, the speed generated can be used at any position. So the arm of a robot can be used at any desired speed,

Table 6: Robotic Arm Distance, Angles and Speed for LINK 1, 2, 3 and 4

S/N	*Robotic Distance (m) Link 1,2,3 and 4*	*Robotic Arm Angle (degree)*	*Robotic Arm Speed (m/s)*
	0.13	1.489	202.1
	0.17	1.948	273.9
	0.69	7.906	287.4
	0.73	8.364	291.9
	2.97	34.030	298.6
	3.01	34.490	305.3
	6.85	78.490	336.8
	6.88	78.830	359.2
	0.09	1.031	381.7
	0.13	1.489	404.1
	0.73	8.364	493.9
	0.78	8.937	538.8

0.82	9.395	583.7
0.86	9.854	718.4
3.832	43.91	763.3
0.03	0.4297	819.4
0.07	1.003	898.0
0.49	7.089	945.1
0.52	7.447	1078.0
1.12	16.04	1123.0
1.15	16.47	1302.0
2.06	29.50	1392.0
2.09	29.93	1527.0
0.03	0.573	1572.0
0.05	0.955	1684.0
0.37	7.066	1841.0
0.39	7.447	1976.0
0.84	16.04	2133.0
0.86	16.42	2245.0
1.54	29.41	2357.0

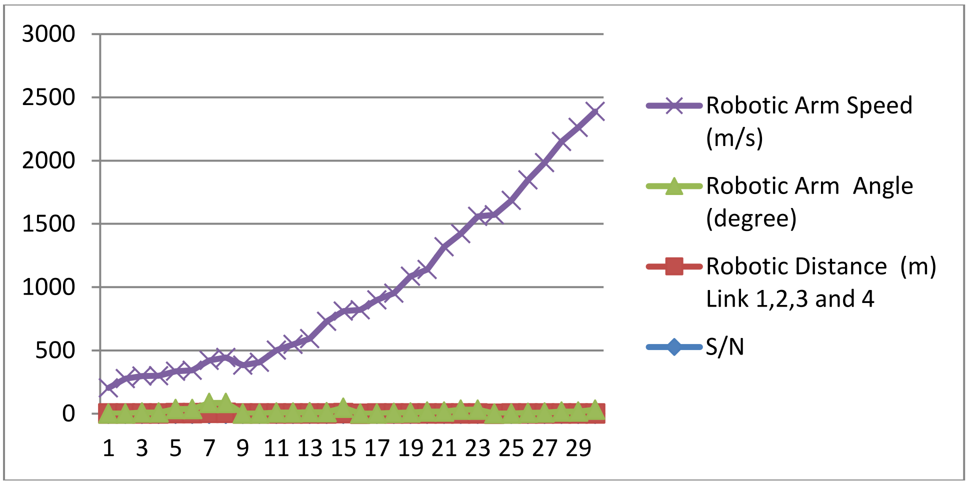

Figure 13: Distance, Angles and Speed Movement

Conclusion

An in-depth study and research have been carried out on robot movement, especially on robot arm movement, A thorough analysis of kinematic and dynamic robot systems was also done by careful mapping of the input-output data in the form of the position and angle of the arm. The multiple data acquired as distance and angles has overcome the limitation placed by the boundary conditions. This has proved that robots can handle multi-task at different positions and angles using the data generated in this paper. This research paper was achieved following these objectives; develop a mathematical model for a robot arm, determine four degrees of freedom of robot arm, determine and develop angles of movement with respect to degrees of freedom, develop a mathematical model for the speed of the robot, generate the voltage and speed of robot arm and develop the entire models of robot arm position and speed

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